Methods of Alternative Projections and "Barycentric method"

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Optimization Class Project. MIPT

Introduction

Today there is developed the convex optimization. But, unfortunately, people can not decide a lot of problems in non-convex optimization. In this work we apply 2 methods of finding "projection" (pseudo projection) onto the Manifold of Stifel, which is not convex, but weakly-convex. This concept is extremely important in the modern optimization.

Manifold of Stifel

Now let us give definition of Manifold of Stifel.

- $S_{nk} = \{X \in R^{n \times k} | X^T X = I\}$, where I - unit matrix.
- It is proved, that S_{nk} - weakly convex set with $R \leq \frac{2}{\sqrt{k^2+3k}}$. Simply put, we can roll ball of radius R on the side of this set without slip.
- It is not difficult notice, that if we transform matrix to the vector, we will get "the long string". Namely, denote $X = (X_1^T, \dots, X_k^T) \in R^{n \times k}$, where X_i - column of original matrix. We can define Manifold of Stifel as $g_{ij} = (X_i, X_j) - \delta ij = 0, 1ijk$, where δij - symbol of Kronecker.

Method of Alternative Projections

It is so difficult to find projection onto non-convex set, but due to S_{nk} -weakly convex and $g'_{ij} \perp g'_{lm}$ if $(i, j) \neq (l, m)$, we can use Method of Alternative Projections to find "projection" onto Manifold of Stifel.The main idea of this method is we gradually make "zero" every g_{ij} (step by step).

Algorithm

Perfom our algorithm.

a $= \max |g_{ij}|$

while $a > 10^{-6}$

if $g_{ij}^a > 0$

while $g_{ij}^a > 0$

$g_{ij}^a = g_{ij}^a - R{g_{ij}^a}'$

then use method of division of section by 2 in order to find $\widetilde{x}$ such $g_{ij}^a(\widetilde{x}) < 10^{-6}$.

if $g_{ij}^a < 0$ we make similar actions in order to find $\widetilde{x}$ such $g_{ij}^a(\widetilde{x}) < 10^{-6}$.

Barycentric method

This method is generalization of previous algorithm. It is worse, because it requires more iterations and more operations of multiplication.

Sphere

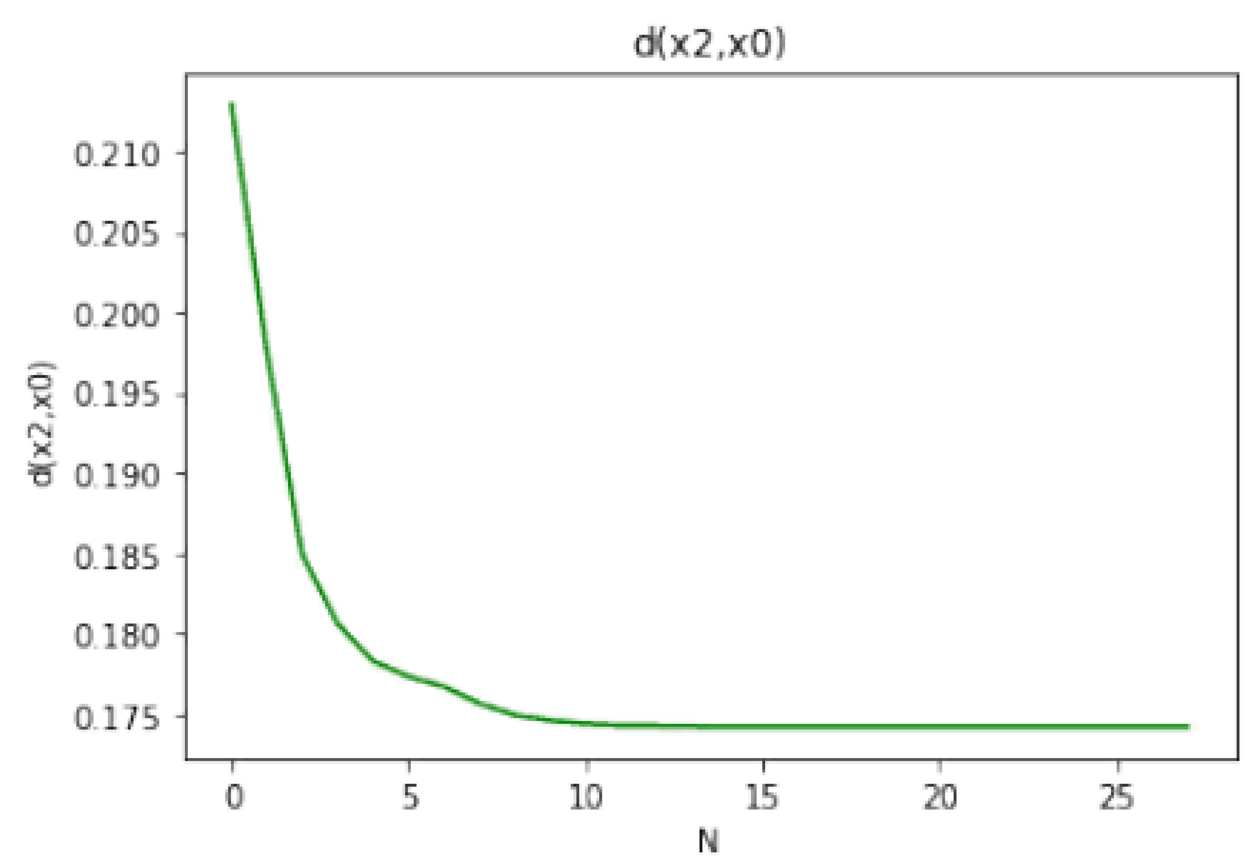
Note, that Manifold of Stefel lies onto the sphere with radius $\sqrt{k}$. If we "go away" from x_0 with manifold normal to the sphere less then R and run the both methods we will see that we get REAL projection. So, there is assumption that these methods give us real projecting

Real projection

We wrote program, which finds the real projection on the manifold of Stefel, using method of Alternative Projection. Now it gives the same results. It is so weird, and we try to find example, when these programs give different results.

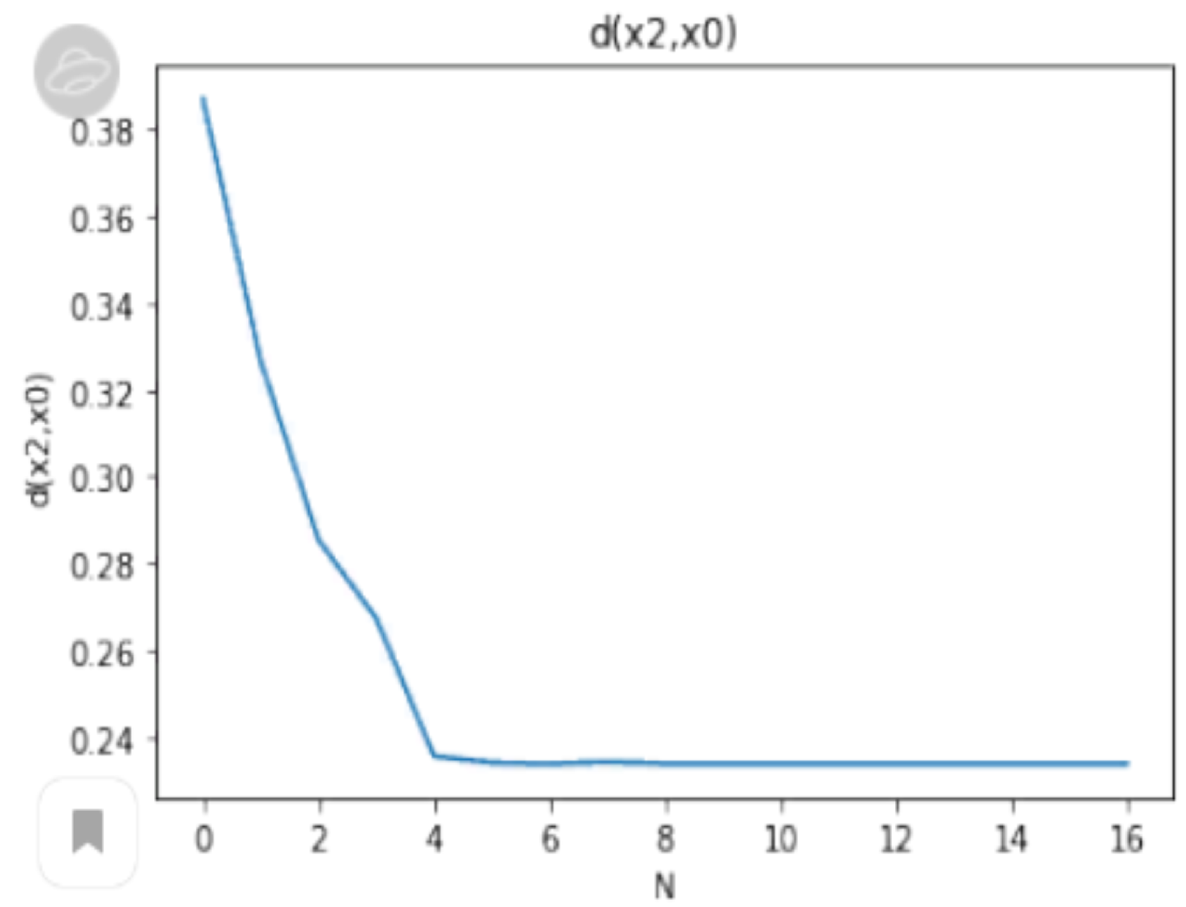
Results of method of Alternative Projection

On this graph, we can see that method right gives the point near with real projection. It is due to Manifold of Stefel contains from ortogonal parts.



Results of Barycentric Method

Also this method converges so fast. But, it requires more operations. Advantage of this method is simple algorithm.



Conclusion

We considered convergence Method of alternative Projections and "Barycentric methods" in case of Manifold of Stifel.It is only start of our work, because now we understand how find real alternative projection without some algorithm and we want to check coincidence its with finding in program. Also we proved that Method of Alternative Projection converges in this case, but so slowly. But, numerical experiments claim that it approaches extra fast and so we want to improve our results

Acknowledgements

This material is based upon work supported by the Vial J, Balashov M.V, Ivanov G.E, Goncharov V.V, Ioffe A.d and Polyak B.T.